NONLINEAR ANALYSIS FOR PILE KINEMATIC RESPONSE

Roberto CAIRO¹, Andrea CHIDICHIMO²

ABSTRACT

The passage of seismic waves through the soil may cause two important kinematic effects on pile foundations: the motion occurring at the foundation level to be different to the free-field motion (with no structure); significant curvatures to be developed by the piles, which can experience considerable bending moments at great depth, not depending on the oscillation of the superstructure. In this paper, a lumped-mass parameter model is used to analyse the kinematic response of the piles. This formulation incorporates p-y curves described by the hyperbolic model along with extended Masing criteria. To validate the proposed approach, comparisons with other theoretical and experimental results are reported.

Keywords: bending moment, superstructure, damping, nonlinear.

INTRODUCTION

In seismic zone, piles are traditionally designed to resist to the inertial forces generated from the oscillation of the superstructure. Recent codes, however, have recognized the importance of the so-called kinematic effects, that arise in the pile from the deformation of the surrounding soil due to the passage of seismic waves. Two primary aspects are peculiar to this kinematic interaction: (1) the earthquake motion observed at the foundation level (named foundation input motion or FIM) differs from the free-field ground motion (soil without the structure), since relatively rigid piles do not follow soil movement; (2) significant strains developed in soft soil by seismic waves induce curvatures and subsequent bending moments (kinematic moments) on piles.

The FIM is usually expressed in the frequency domain in terms of the kinematic response factors, relating the absolute values of pile head displacement to the free-field horizontal component at ground surface (Kaynia and Kausel, 1982; Gazetas, 1984; Mamoon and Banerjee, 1990; Fan et al., 1991; Gazetas et al., 1992; Kaynia and Novak, 1992). In general, at lower frequencies the piles essentially follow the movement of the ground, while at higher frequencies they experience considerably reduced deformations, as a direct consequence of the incompatibility between the seismic wave pattern in the soil and the relative rigidity of the piles. In the case of fixed-head piles, a rotational component of motion arises, which exhibits several peaks at the natural frequencies of the soil deposit. These peaks tend to be reduced substantially for stiffer piles, which undergo smaller displacements, as mentioned above.

The kinematic moments developed in the pile have received less attention compared to the seismic response of the pile head. Nevertheless, analytical and field evidences (Mizuno, 1987; Tazoh et al., 1987; Mylonakis, 2001; Nikolaou et al., 2001) identify kinematic bending moments to be a potential cause of pile damage when associated to the presence of strong discontinuities in stiffness in the soil profile. Available methods

¹ Researcher, Faculty of Engineering, University of Calabria, e-mail: cairo@dds.unical.it
² Engineer, Faculty of Engineering, University of Calabria.
for evaluating deformations and internal forces of piles under seismic loading are mainly based on numerical procedures. The finite element (Wu and Finn, 1997; Cai et al., 2000; Kimura and Zhang, 2000) and the boundary element methods (Kaynia and Kausel, 1982; Mamoon and Banerjee, 1990; Guin and Banerjee, 1998; Cairo and Dente, 2007) provide the most rigorous technique to analyse soil-pile interaction. The first permits to take into account nonlinearity and heterogeneity of the soil, although it is very expensive from a computational point of view. On the contrary, the second is more efficient but restricted to model linear behaviour of the soil-pile system, in principle. Among simplified approaches, the methods based on the use of $p-y$ curves (Kagawa and Kraft, 1980a, 1981; Nogami et al., 1992; El Naggar and Novak, 1996; Boulanger et al., 1999; El Naggari et al., 2005; Cairo et al., 2008) reveal quite accurate despite the modest computational effort. These curves relate the applied pressure to the soil at different depths along the pile to the soil deformation at the same depths. The use of $p-y$ curves can be easily incorporated into a Winkler model or a lumped-parameter model as will be described in the following section.

In this paper, soil-pile kinematic interaction is analysed by means of a lumped-mass parameter model. The soil is assumed to behave as either a linear viscoelastic medium or a nonlinear material, with the use of $p-y$ curves based on the hyperbolic model. The analyses focus on the influence of soil layering, the presence of the superstructure and the effects of radiation damping on the bending moments along the pile.

**METHOD OF ANALYSIS**

For practical applications the lumped-mass parameter model represents a powerful tool to study quite complicated foundations. In this framework, seismic excitation can be processed directly in the time domain and nonlinear behaviour of the system can be easily taken into account by updating the stiffness matrix of the model at each time increment. The *driving* loads produced by ground motion and applied to the masses must first be determined analysing the soil model only (free-field). Fig. 1 shows the lumped-mass parameter model used in this study to represent a single pile supporting a shear-type single-degree-of-freedom structure and connected to the surrounding soil by springs and dashpots, which provide the interaction forces in the lateral direction. Free-field response of the soil is evaluated by means of an analogous model corresponding to a horizontally layered deposit excited at its base by vertically propagating shear waves.

![Figure 1. Lumped-mass parameter models adopted.](image-url)
The dynamic equation of motion of the pile-soil system considered assumes the following form:

\[
[M_p] \{\ddot{y}\} + [K_p] \{y\} + \{p\} = 0
\]  

where \([M_p], [K_p]\) = mass and stiffness matrices of the pile (Chopra, 2007), respectively; \(\{\ddot{y}\}\) = vector of the total accelerations of the lumped masses; \(\{y\}\) = vector of their lateral displacements; \(\{p\}\) = vector containing soil reactions. At the generic depth, soil reaction can be split in two components which can be written as:

\[
p = p_1 + p_2 = k (y - y_f) + c (\dot{\delta} - \dot{y}_f)
\]

in which \(y_f\) and \(\dot{y}_f\) are the free-field displacement and velocity of the soil at prescribed time, respectively; \(k\) is the stiffness of the springs, and \(c\) the dashpots coefficient. For a linearly elastic soil, the spring coefficient is formulated as:

\[
k = \delta E_s
\]

being \(E_s\) the soil modulus and \(\delta\) the average soil-reaction parameter which depends on soil-pile flexibility, soil profile and loading conditions. It is worth noting that \(E_s=2(1+\nu_s)G_0\), where \(\nu_s\) = Poisson’s ratio, and \(G_0\) = small-strain shear modulus. Typical values of \(\delta\) are between 0.8 to 1.5 (Yoshida and Yoshinaka, 1972; Liou and Penzien, 1977; Roësset and Angelides, 1979; Kagawa and Kraft, 1980b; Dobry et al., 1982), even though significant difference in \(\delta\) can be found due to the different procedures used. Kavvadas and Gazetas (1993) proved that, contrary to pile displacements, pile bending moments reveal some sensitivity to this parameter, which can attain values up to 4 in heterogeneous soils. In this work the following relation is adopted:

\[
\delta = \frac{\alpha}{1-\nu_s^2} \left( \frac{E_s d^4}{E_p I_p} \right)^{1/12}
\]

in which \(d\) = pile diameter, \((E_p I_p)\) = pile flexural rigidity. Equation 4 has been derived from the original formula of Yoshida and Yoshinaka (1972), which is based on the theoretical solution for a beam on a homogeneous elastic medium. The parameter \(\alpha\) is introduced to take into account the amplification of the dynamic load on the pile due to the change of stiffness at the layer interface for a two-layer soil deposit, and can be calculated as:

\[
\alpha = \left( \frac{L}{d} \right)^{2/5} \left( \frac{H_1}{H_2} \right)^{\frac{1}{15}} \left( \frac{V_{s1}}{V_{s2}} \right)^{1/4}
\]

where \(L\) = pile length; \(H_1, H_2\) = thickness of the upper and lower layer, respectively; \(V_{s1}, V_{s2}\) = shear wave velocity in the upper and lower layer, respectively.

The dashpot coefficient \(c\) is generally used to account for radiation damping (Berger et al., 1977; Dobry et al., 1982; Gazetas and Dobry, 1984). Moreover, it can include viscous damping at small strains, which is not considered in many constitutive nonlinear models, such as the hyperbolic or the Ramberg-Osgood
models. In the present study the damping coefficient $c$ is formulated according to Gazetas and Dobry (1984) and incorporates a viscous component as a function of the spring stiffness $k$ (Kavvadas and Gazetas, 1993) by the following expression:

$$c = 2\rho_s V_s d \left[ 1 + \left( \frac{3.4}{\pi(1 - \nu_s)} \right)^{3/4} \right] \left( \frac{\pi}{4} \right)^{3/4} \left( \frac{\omega d}{2V_s} \right)^{-1/4} + 2\xi_s \frac{k}{\omega}$$

(6)

in which $V_s$ = shear wave velocity, $\xi_s$ = damping ratio of the soil, $\rho_s$ = mass density of the soil, $\omega$ = circular frequency of the motion.

Denoting with $[K_s]$ and $[C_s]$ the diagonal matrices containing the spring coefficients and the damping coefficients of the soil, respectively, the dynamic equilibrium equation 1 can be written as:

$$[M_p] \{ \ddot{\delta} \} + [C_s] \{ \dot{\delta} \} + ([K_p] + [K_s]) \{ y \} = [K_s] \{ y_g \} + [C_s] \{ \dot{\delta}_g \}$$

(7)

which can be solved numerically at each time step using the Newmark method (1959).

To consider the nonlinear behaviour of the soil, the hyperbolic model with extended Masing rules described by Phillips and Hashash (2009) is adopted. The backbone curve is defined as:

$$\tau = \frac{\gamma G_0}{1 + \beta \left( \frac{\gamma}{\gamma_r} \right)^s}$$

(8)

where $\tau$, $\gamma$ = shear stress and shear strain, respectively; $\gamma_r$ = reference shear strain; $\beta$, $s$ = dimensionless factors which can be determined by curve fitting of laboratory test data.

The dashpot coefficient is taken as independent of strain level since the effect of hysteretic damping induced by nonlinear soil behaviour is captured through loading-unloading cycles in the constitutive model adopted. The influence of radiation damping on the pile response will be discussed in the following. The stiffness matrix $[K_s]$ in equation 7 is updated at each time increment to incorporate nonlinearity of the soil according to the hyperbolic model assumed.

**COMPARISONS WITH EXISTING RESULTS**

In order to validate the presented approach, comparisons with other theoretical and experimental results are reported in the following. A single fixed-head pile with length $L=20$ m, diameter $d=0.6$ m, and Young’s modulus $E_p=2.5 \times 10^7$ kN/m$^2$ is first considered (Fig. 2). The pile is embedded in a two-layer soil deposit resting on a stiffer bedrock. The thickness of the layers has been varied and soil stiffness contrast has been changed as a function of the shear wave velocities in the two layers. This system corresponds to the reference scheme adopted within the RELUIS project, a major research activity carried out by a consortium of Italian universities (Maiorano et al., 2007; Sica et al., 2007; Cairo et al., 2008) committed to investigate soil-pile kinematic interaction. Input motions have been selected from the SISMA database of Italian seismic events (Scasserra et al., 2008), scaled to 0.35 g and applied to the bedrock level.
Fig. 3 shows the envelopes of the maximum bending moment along the pile for the case with $H_1=H_2=15$ m, $V_{s1}=100$ m/s and $V_{s2}=400$ m/s. The soil is assumed to behave as a linear elastic medium. On the basis of the equivalent shear wave velocity $V_s(=160$ m/s), the soil profile can be classified as ground type D, in which, according to Eurocode 8, kinematic effects should be considered in the design of pile foundations. These results have been obtained considered two strong Italian seismic events: the 1976 Friuli earthquake (the NS component labelled as A-TMZ000 and the EW component A-TMZ270) and the 1980 Irpinia earthquake (the NS component labelled as A-STU000 and the EW component A-STU270).

The kinematic moments calculated with the proposed method compare favourably with those computed with the rigorous BEM approach developed by Cairo and Dente (2007). As can be noticed, the maximum bending moment is always found in correspondence to the soil layer interface. As suggested by Mylonakis (2001), if we use bending strain at the outer fibre of the pile cross-section, i.e.:

$$\varepsilon_p = \frac{M}{E_p I_p} \frac{d}{2}$$

(9)

to quantify pile damage, in the case plotted in Fig. 3, both the A-STU000 and A-TMZ270 records produce peak bending strains of about 0.12%, which are greater than the typical value (0.1%) that can be considered to be enough to produce damage in conventionally-designed concrete or steel beams under static loading conditions. As supported by the observations carried out by Cairo et al. (2009), this is due to the fact that the two accelerograms are characterized by mean periods that are close to the fundamental period of the soil deposit.

The influence of the superstructure on pile bending is illustrated in Fig. 4. In this connection, as schematized in Fig. 1, a single-degree-of-freedom oscillator with period $T=0.2$ s is considered. The solution for the pile without superstructure is plotted again and referred to as kinematic interaction. As can be observed, the magnitude of the bending moment at the pile head increases and reaches by far the maximum value (thick line). On the contrary, the bending moment at the layer interface remains practically unchanged. These
results reveal the dominant effect of kinematic interaction at the depth where the stiffness of the soil varies abruptly. These observations are in accordance with previous theoretical (Kaynia and Mahzooni, 1996; Kaynia, 1997) and experimental results (Dihoru et al., 2009; Moccia, 2009).

Figure 3. Envelopes of the bending moment along the pile for different seismic motions.
In the same figure, the solutions obtained without considering radiation damping in equation 6 are also shown for both kinematic interaction (circle) and pile with superstructure (triangle). As can be noticed, radiation damping affects bending moment only when the superstructure is present and along the upper part of the pile. However, it should be pointed out that the extent of this trend may be influenced by the features of the seismic motion considered (Conte and Dente, 1988).

Figure 4. Influence of superstructure and radiation damping on pile bending moments.

Figure 5. Acceleration-time histories at pile head and ground surface (free-field) for A-STU000 seismic record.
Finally, the calculated horizontal acceleration-time histories for A-STU000 record at the pile head and at ground surface in free-field condition are plotted in Fig. 5. As expected, the acceleration values at the pile head are significantly smaller than those at the ground surface.

Comparisons with experimental results
The theoretical results obtained using the proposed approach are compared with dynamic centrifuge model tests of a single-pile-supported structure documented by Wilson et al. (1997). The soil profile consisted of a soft clay layer 6.4 m thick overlying a dense sand 11.1 m thick. The aluminum pile used in the experiment is approximately equivalent to a steel pipe pile with a diameter of 0.67 m and wall thickness 19 mm. The total length of the pile was 20.57 m. The superstructure was a mass of 49.1 Mg attached at 3.81 m from the ground surface. The input motion applied at the base of the system was a strong motion record from Port Island in the 1995 Hyogoken-Nambu (Kobe) earthquake scaled to 0.055g (event B), 0.016g (event C), 0.20g (event D) and 0.58g (event E). The layout of the pile and the soil profile considered in the calculations are presented in Fig. 6. The characteristics of the soil layers have been determined according to the indications provided by Boulanger et al. (1999). The parameters requested to define the backbone curve of equation 8 have been got from Phillips and Hashash (2009).

The calculated and recorded peak bending moment distributions along the pile for shaking events B, C, D and E are shown in Fig. 7. The theoretical results obtained by Boulanger et al. (1999) and El Naggar et al. (2005), both based on nonlinear Winkler formulations with p-y curves, are also plotted.

In general, the bending moments calculated with the proposed method tend to be conservative with respect to the measured values, especially for event B. On the whole, the performance of this solution is fairly satisfactory and the comparison with the other theoretical approaches is favourable.

CONCLUSIONS
A simple lumped-mass parameter model for evaluating the kinematic response of single piles in the time domain has been presented. The approach involves a preliminary site response analysis, that can be attained using the same theoretical basis. The soil-pile interaction is taken into account by p-y relations which can simulate both linear and nonlinear soil behaviour. At this end, the hyperbolic model with extended Masing rules as described by Phillips and Hashash (2009) has been used. Nonlinear analyses reveal quite efficient from a computational point of view, since they can be performed very easily by updating at each time step.
the stiffness matrix of the soil system. Comparisons with both theoretical and experimental existing results have assessed the validity of the approach. The analyses have pointed out the importance of considering soil-pile kinematic interaction in layered soils with sharp stiffness contrast when bending moments are calculated, even if the superstructure is present. On the contrary, pile head motion always reveals smaller than the free-field ground motion. The influence of radiation damping can be found only in the presence of the superstructure and in the upper part of the pile.

Figure 7. Pile maximum bending moment.
REFERENCES


